The Value of Netting

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Abstract

Close-out netting is a standard procedure in derivatives markets that protects counterparties' claims on derivatives contracts. Despite the ubiquituous nature in derivatives markets, the literature has not studied counterparties' motivation to use close-out netting. This paper aims to fill this gap. I develop a theoretical model and interpret close-out netting as the option to make a derivative state-dependent. I show that firms use close-out netting as a tool to transfer risk from derivatives counterparties to creditors. Whether the risk-transfer is sub-optimal, however, depends on the type of the derivative. In the case of additional risk sources that make the derivative an imperfect hedge, close-out netting can protect firms' creditors from counterparty risk. The findings of this paper are in line with prior evidence on derivatives contracts and their use in bankruptcy cases.

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1 Introduction

Derivatives markets are huge: at the end of December 2019 the total gross market value of over-the-counter (OTC) derivatives amounted to almost $12 trillion worldwide according to the Bank for International Settlements (2020). The outstanding notionals even reached pre-crisis levels with close to $650 trillion at the end of June 2019. Apparently, derivatives enjoy a lasting popularity, despite the experiences of the financial crisis a little more than ten years ago. Due to the enormous size and interconnectedness of OTC markets, a major concern is that the default of one key player could trigger another financial crisis as in 2008. In order to manage the risk associated with the default of a counterparty - the so-called counterparty risk - market participants use a variety of tools. One of those measures is close-out netting.

Netting can take place in two ways in derivatives markets: payment netting and close-out netting. Payment netting is used by counterparties who need to exchange payments, e.g. variation margins. The counterparties sum up all claims and liabilities which should be settled at a specific point in time, e.g. at a specific day, and exchange a single net payment (Gregory (2014), p. 60). Because regulatory advancements after the 2008 crisis have implemented mandatory day-to-day margining for wide ranges of OTC products, nowadays payment netting has rather become a way to process payments more easily and is less effective in reducing counterparty risk.

Close-out netting comes into play when one of the counterparties defaults and, hence, is unable to honor its contractual obligations on (a part of) the existing derivative contracts. Essentially, close-out netting consists of two processes. First, the contracts are closed out which means that, given the default of a counterparty, all contracts are terminated and valued at current market prices. Then, similar to payment netting, the surviving counterparty can offset claims with liabilities against the defaulted counterparty (see Mengle (2010)). If the resulting net position is a liability, the non-defaulting entity has to provide the payment, while the entity becomes a creditor if she has a net claim. The goals of close-out netting are, firstly, to minimize the time and scope of uncertainty related to contracts with defaulted entities and, secondly, to prevent large losses in the balance sheets of the non-defaulting derivatives counterparties.

Starting with the US Bankruptcy Reform Act of 1978, close-out netting has gradually gained importance in international derivatives regulation. For the first time, the US Bankruptcy Reform Act of 1978 allowed setoffs for a very limited number of financial products and contracts. More specifically, it exempted such setoffs from the standard procedure of bankruptcy law, especially the automatic stay, which prohibits any outflows from defaulted entities (Schwarcz and Sharon (2014), p. 1724ff). In the following years, several amendments and acts expanded this initial exemption to more and more products and entities, and added exemptions for close-outs such that close-out netting took the overarching form it has today (see Schwarcz and Sharon (2014)). Internationally, the US path to close-out netting served as a blueprint for derivatives regulation. Shortly after the 1990 Amendment, the European Community introduced close-out netting into European law in the form of the directive 96/10/EC.[1] Basel III, which has been crafted into European Law by the Capital Requirements Regulation in 2013, poses the latest update on international standards on close-out netting.[2] The main novelty has been that close-out netting leads now to lower capital requirements.

In the light of the legal status close-out netting has gained over time, one can argue that the size of derivatives markets decoupled from the associated risks (see Mengle (2010), p. 1). After the financial crisis, regulatory authorities tried to widen the scope of close-out netting even further by introducing mandatory central clearing. Thereby, a lot of derivative classes have been shifted to central counterparties (CCPs) which take on the counterparty risk by stepping in the middle of derivative contracts - a process that is called "novation" of a contract (see Gregory (2014)). The idea is that a key node, that has the information about all trades and, hence, can net all trades of a counterparty, is able to minimize the exposure in case of a defaulting market participant. The question whether mandatory central clearing actually achieved this goal is subject to an ongoing debate.

Still, the effects of close-out netting are significant. At the end of December 2019, for example, the gross credit exposure, which compared to the gross market value reflects the scope of netting possibilities, totaled to only $2.4 trillion, i.e. approximately only 20 percent of the market size of derivatives markets (see Bank for International Settlements (2020)).

The question arises what drives (financial) firms to use close-out netting as a standard tool in bankruptcy cases. This paper tries to analyze potential reasons by modeling close-out netting and its impacts theoretically. For this purpose, I extend an existing model by Bolton and Oehmke (2015) which combines incomplete debt contracts with derivatives markets. Compared to Bolton and Oehmke (2015), I add both a contractual definition of close-out netting and mutual counterparty risk on derivatives markets. Close-out netting gives derivatives counterparties the possibility to manage their portfolio better, especially in the case of default of a counterparty. For example, firms now have the possibility to exit a position by taking on a reverse position with the same counterparty and agreeing to net. In our model, close-out netting will allow to write derivatives contingent on a larger state space than before. This will lead to derivative contracts being complete while debt contracts still remain incomplete. Hence, derivatives enjoy an advantage compared to other financing instruments.

The model consists of a firm that has access to a profitable but risky investment project, yet has no own funds. Therefore, the firm must access debt markets to finance the investment. Because debt contracts are incomplete, i.e. the contract does not specify payments for all states of the world, the firm still faces default risk which can be hedged with derivatives offered by a counterparty. If close-out netting is not possible, the derivative contract suffers also from incompleteness while close-out netting makes derivatives complete. Derivatives counterparties are subject to capital requirements and other

1 See Publications Office of the European Union (1996).

2 See Basel Comitee on Banking Supervision (2004) and Publications Office of the European Union (2013).

regulations which lead to a deadweight cost proportional to the maximum exposure of the counterparty. Moreover, counterparties can default for exogenous reasons.

I find that the effect of close-out netting depends on the type of derivative. If the derivative is contingent on the revenues of the firm, close-out netting leads at best to the same outcome as a contract without close-out netting. Especially, if the firm cannot commit to its derivatives portfolio, close-out netting induces a risk-transfer from derivatives counterparties to creditors which is sub-optimal because the risk-transfer increases the cost of debt for the firm and, hereby, the overall deadweight cost. However, close-out netting helps to avoid inefficiencies if the derivative is not a perfect hedge for the cash flow risk, e.g. when the derivative is defined on a basis that is not perfectly correlated with the cash flows of the firm. Then, the firm can use close-out netting to negotiate a contract which ensures that the counterparty always pays when the firm is in need of additional funds. This holds true even when the firm suffers from limited commitment. In this situation a risk-transfer might not only be individually rational but also socially beneficial.

Besides this main finding, the paper confirms the general direction of the results of Bolton and Oehmke (2015) and extends their analysis to counterparty risk on the side of the derivatives counterparty. This counterparty risk limits the firm's incentives to use derivatives as a hedging tool. There are two effects of counterparty risk. First, counterparty risk makes the derivative less reliable to ensure the firm's continuation after a low cash flow which, in turn, increases the cost of debt. Secondly, this increased cost of debt leads to greater deadweight costs and reduces the firm's profits. This result might offer an explanation why small firms go to large dealers with lower default risk.

Related Literature. This paper touches two strands of literature. On the one hand, it discusses the effects of close-out netting in a theoretical framework, and, on the other hand, it features the well-known problem of risk-shifting which arises in incomplete contracting frameworks.

In the past years netting has received notable attention by scholars motivated by the growth of OTC markets before the financial crisis in 2008 and the renewed interest of policy makers in OTC derivatives markets - especially the introduction of mandatory central clearing via CCPs.[3] Because the industry claims that netting is a key feature to foster the stability of financial markets, the research on netting has focussed on the effects of netting within derivatives markets and on creditors outside derivatives markets.

A significant part of the literature on netting deals with the question whether bilateral or multilateral netting is better suited to reduce counterparty risk. Duffie and Zhu (2011) introduce the concept of "netting efficiency" - the reduction in credit exposure - in order to measure the usefulness of different netting arrangements. Their main results establish that in a market with N participants and K asset classes either the number of clearing members or their respective exposure in the cleared asset class

3 From 1998 to the eve of the financial crisis in 2008, the total notional outstanding of derivatives has grown to more than sixfold (see Cont and Kokholm (2014)). The same holds true for total gross market values.

must be sufficiently large in order to make the introduction of mandatory central clearing for one asset class beneficial.[4] Put differently, there exists a trade-off between bilateral and multilateral netting. Primarily, close-out netting allows derivatives counterparties to reduce their counterparty risk substantially. However, if some derivatives classes are centrally netted while other derivatives classes are not, the multilateral netting across all market participants in some derivatives class destroys the bilateral netting possibilities across derivatives classes. The risk-reduction of netting is strongest when the market participants use the maximum scope of netting possibilities which can be effectively done by a universal CCP. This universal CCP would pose the central node in the OTC market which clears and nets all trades.[5] In a quantitative study, Lewandowska (2015) confirms the main results of Duffie and Zhu (2011) and adds a threshold for the minimum number of cleared asset classes. Cont and Kokholm (2014) deviate from the assumption that asset prices are independently distributed and show that the "thresholds for central clearing" are highly sensitive to the distributional assumptions on the derivatives classes, e.g. the correlation across asset classes plays an important role. Kubitza et al. (2019) mainly examine the effects of systematic risk, extreme market stress and the impact of loss allocation of a CCP on the thresholds for central clearing and the individual market participant's incentives to centrally clear. They find that systematic risk increases the threshold for central clearing dramatically and, under extreme events, a suspension of central clearing is beneficial. Moreover, they show that loss allocation mechanisms by the CCP explain why some market participants are reluctant to central clearing.

In general, numerous studies have stressed the risk reduction argument for derivatives counterparties within OTC markets, e.g. see Bliss and Kaufman (2006), Pirrong (2009) and Pirrong (2013). However, these studies also argue that close-out netting remains not without consequences for outside creditors. Because netting effectively reduces the bankruptcy estate, the risk reduction for derivatives counterparties comes at the cost of greater risk for outside creditors. Hence, netting induces a risk-transfer from derivatives counterparties to other creditors such as debt holders, which usually enjoy the highest priority in the insolvency process. Put differently, netting is a wealth transfer from outside creditors to derivatives counterparties in the case of insolvency.

Due to the ambiguous effects of netting, some papers discussed the overall effect of netting on financial stability and social welfare.[6] Pirrong (2009) notes that this risk transfer is not necessarily bad. If the risk is better borne by the outside creditors or the preferences of the involved agents lead to an overall improvement in welfare, netting will foster financial stability. Bolton and Oehmke (2015)

4 Such single-asset class CCPs actually exist currently in the clearing market. For example, ICE Clear Europe has distinct clearing services specialized on commodity derivatives and CDS while ICE Clear Netherlands specializes on equity derivatives.

5 Although the advantages of a universal CCP are quite intuitive and even go beyond the benefits of close-out netting, in reality, this idea faces several legal and operational challenges. Moreover, a universal CCP is also not the panacea to counterparty risk. For a discussion, see Duffie and Zhu (2011), Cont and Kokholm (2014) and Kubitza et al. (2019).

6 For the sake of brevity, this literature review only encompasses the economic papers concerning the impact of netting on financial stability. However, this issue has been discussed extensively by law scholars for the case of US bankruptcy law. For a general discussion of legal exemptions for derivatives (featuring close-out netting), see, for example, Lubben (2010), Claor (2013) and Schwarcz (2015).

contradict this statement and show in an incomplete contracting environment that a risk transfer from counterparties to outside creditors reduces overall welfare, if all agents are risk-neutral.[7]

Besides the overlaps with the literature on netting, this paper builds on the literature on the multiple possibilities for (financial) institutions to dilute creditors in an incomplete contracting environment.[8] In Brunnermeier and Oehmke (2013), firms cannot commit to the maturity structure of their external financing, i.e. the debt contracts are incomplete with respect to the maturity. This induces an inefficient shortening of maturities because each creditor anticipates that the firm would use short-term rollover debt to dilute the long-term creditors. As this dynamic leads to additional risk and underinvestment, the "maturity rat race" is inefficient. Bolton and Oehmke (2015) show that firms which cannot commit to their derivative contracts use legal advantages of derivatives in order to engage in speculative derivatives trading. Lastly, Donaldson and Piacentino (2019) have shown a possible mechanism in the banking sector. Banks use mutual debt holdings in order to have creditors, namely the other banks, which can be diluted by issuing more senior debt in later periods when liquidity shocks arise.

To sum up, all of these papers feature a risk-shifting across creditors because an institution cannot fully commit to all future actions, e.g. by writing a complete financial contract. Even though this risk-shifting is in favour of the shareholders, it leads to overall suboptimal outcomes and seems undesirable from a pollicy perspective.[9]

The remainder of this paper is organized as follows. Section 2 describes the model setup. In Section 3 a baseline of the model without derivatives is derived. Section 4 solves the model in the absence of netting and Section 5 when the derivative contracts allow for close-out netting. In Section 6 some extensions and their implications are discussed. Finally, Section 7 summarizes the analysis and hints at potential extensions for future work.

2 Model Setup

The model extends the framework of Bolton and Oehmke (2015) in two dimensions. First, the counterparty suffers from an exogenous default probability. Secondly, the main focus does not lie on the bankruptcy treatment of the derivative contract but the state space the contract can be defined on. In this section, the model of Bolton and Oehmke (2015) is presented with the extensions outlined above.

7 Besides these considerations regarding the overall welfare, close-out netting could contribute to systemic risk according to Bergman et al. (2004) because netting arrangements usually encompass triggers which cannot reliably distinguish between insolvency and illiquidity issues, e.g. missed margin calls.

8 The incomplete contracting framework was mainly developed by the works of Grossman and Hart (1986), Hart and Moore (1990), Bolton and Scharfstein (1990), Aghion and Bolton (1992), Hart and Moore (1994), Bolton and Scharfstein (1996) and Hart and Moore (1998).

9 Note that this result depends highly on the preferences of the model's agents. As discussed above, there are also other views on this issue, e.g. Pirrong (2009).

Consider an economy which lasts three periods, $t \in \{0, 1, 2\}$, and consists of three agents: a firm, a creditor and a counterparty. For simplicity, the risk-free interest rate is normalized to zero.

2.1 The firm

The firm is only limited liable and has access to a risky investment project with positive net present value. The investment needs a fixed cost of $F > 0$ at $t = 0$, yields a stochastic payoff C_1 at $t = 1$ and, upon continuation until the last period, a safe payoff C_2 at $t = 2$. Specifically, in $t = 1$ with probability θ the investment yields a high payoff $C_1^H > 0$, while with probability $1 - \theta$ it only pays out a low cash flow $0 < C_1^L < C_1^H$. The last-period payoff $C_2 > 0$ is only realized when the firm continues its operations until the last period. The firm can be thought of as any typical firm which faces business risk stemming from its operations, e.g. an airline company, which has unexpected shocks to its cost due to increasing oil prices, or a bank, which potentially faces mortgage losses, etc.

The firm has no own funds and, hence, must access external financing from competitive debt markets. It borrows the necessary initial outlay F from a risk-neutral creditor, who is endowed with deep pockets. That is, the creditor faces no wealth constraint and supplies the firm capital in a standard incomplete debt contract as described in Bolton and Scharfstein (1996) (p. 5).[10] In return for the initial outlay of F, the firm is contractually obliged to repay R at date 1. If the firm fails to make the repayment, all rights of the firm are transferred to the creditor who cannot operate the firm but only has the option to liquidate the project and the firm. Moreover, the firm suffers from a limited pledgeability problem: despite being observable, neither C_1^H nor C_2 are pledgeable. That is, the firm could run away with these funds at date 1. The creditor will not renegotiate the debt contract and always liquidate the firm after a strategic default.

In order to make the financing arrangement between the firm and the creditor non-trivial, I assume $F > C_1^L$. In the high cash flow state, however, the firm will always be able to repay the creditor, i.e. C_1^H always suffices to honor all claims against the firm. The last-period (expected) cash flow or continuation value C_2 makes the firm risk-averse if C_2 is large enough. Therefore, I assume that C_2 is sufficiently large such that the firm has an incentive to continue operations until the end.

2.2 The counterparty

At date 0, the firm has access to competitive derivative markets with risk-neutral counterparties. A derivative is a one-period contract which specifies transfers τ contingent on some underlying and, if possible, additional parameters. More specifically, τ^B denotes a transfer *from* the counterparty *to* the

10 In this setup, the incompleteness of the debt contract can be modeled by assuming that R cannot be made contingent on the realization of the cash flow or, put differently, that the investment project is sufficiently complex such that it is not feasible to write a complete contract. An alternative explanation would be that the "cash flow is 'observable' but not 'verifiable'" (Bolton and Scharfstein (1996), p. 5). The different origins of incompleteness of (financial) contracts have been discussed extensively by Grossman and Hart (1986), Hart and Moore (1990), Aghion and Bolton (1992) and many others.

firm - the superscript B for the firm as the buyer of the insurance -, while τ^S denotes a transfer *from* the firm *to* the counterparty - the superscript S for the counterparty as the seller of the insurance. Therefore, due to feasibility and optimality considerations, I have

$$-\tau^B = \tau^S \tag{1}$$

for all states the contract is defined on. One can think of the derivative contract as a representation of a whole portfolio of derivatives between a firm and a counterparty. Figure 1 depicts the full contractual setup in $t = 1$, i.e. when both the derivative and the debt contract are settled.

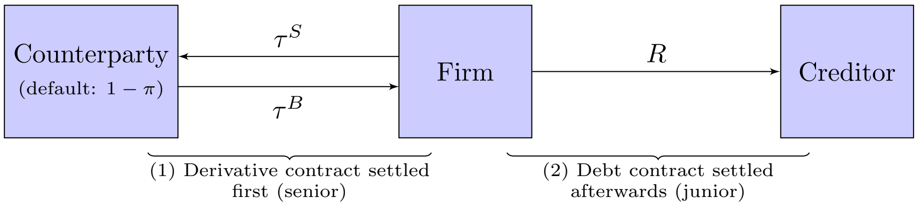

Figure 1: Contractual setup in $t = 1$.

I assume that the derivative is senior to the debt contract in the case of bankruptcy, i.e. the firm serves the counterparty's claims before serving the debt contract. This assumption reflects the favourable treatment of derivatives in the insolvency process - for the treatment in German law, see Braun et al. (2014).[11]

Financial institutions who offer derivative contracts typically face certain obligations associated with this exposure. For example, Basel III prescribes additional capital requirements for derivatives dealers, and counterparties have to post margin on a frequent basis.[12] To capture these costs, the counterparty incurs deadweight costs proportional to the maximum exposure the counterparty can face. The maximum exposure corresponds to the maximum transfer τ^B to the firm, i.e. according to (1) the minimum of τ^S, in any state. The deadweight cost are represented by a per-unit cost $\delta > 0$.[13]

Despite the additional capital requirements, the counterparty is not "rock-solid". Only with probability π it stays solvent at date 1, while it defaults with probability $1 - \pi$ no matter what cash flow

11 The assumption that derivatives are senior is simplifying. According to insolvency law, claims from derivatives are usually unsecured claims. However, there are a number of legally recognized instruments which ensure a higher priority for derivatives. For example, payments on derivatives are exempt from the automatic stay and collateral posted on derivative positions can be seized immediately. Today, these practices are internationally accepted by all important legislations - according to some critics, also because of the relentless efforts of the International Swaps and Derivatives Association (ISDA), see, for example, Schwarcz and Sharon (2014) and Schwarcz (2015). Furthermore, the introduction of (mandatory) central clearing after the financial crisis 2008 and the related operational rules such as daily exchange of margins, mandatory initial margins, etc. have build another layer of safety for derivatives counterparties. For more information on the legal treatment of derivatives, see Bliss and Kaufman (2006) and Bolton and Oehmke (2015). For more information on the activities of central counterparties, see Gregory (2014).

12 For further information, see Basel Comitee on Banking Supervision (2011) and Publications Office of the European Union (2013).

13 For a microeconomic foundation of the collateral cost, one could model the deadweight cost as the result of a moral hazard problem as is done in Biais et al. (2016) and Bolton and Oehmke (2015).

state the firm is in. In the case the counterparty defaults she cannot make any payments but still values incoming transfers the same way as if she stayed solvent. If the counterparty stays solvent, she is able to serve any claim against her, i.e. she has deep pockets.

2.3 Close-out netting

I model close-out netting as a legal tool which allows to extend the state space of the derivative contract. More specifically, a derivative contract without close-out netting specifies the transfers τ^B and τ^S contingent on the first-period cash flow $C_1 \in \{C_1^L, C_1^H\}$. This basic derivative contract corresponds to the definition of a derivative contract in Bolton and Oehmke (2015) in the absence of basis risk. When close-out netting is possible the firm and the counterparty can specify transfers contingent on the first-period cash flow and the solvency of both counterparties, i.e. the state space of the derivative contract is

$$(C_1, D_F, D_C) \in \{C_1^L, C_1^H\} \times \{0, 1\} \times \{0, 1\}, \tag{2}$$

where $\{D_F = 1\}$ describes the event that the firm defaults and $\{D_C = 1\}$ the event that the counterparty defaults. For example, $\tau^B(C_1^H, 0, 1)$ is the transfer from the counterparty to the firm after C_1^H when the firm stays solvent while the counterparty defaults.[14] Hence, close-out netting allows to finetune the contract for the states where either the firm or the counterparty default.

The firm and the counterparty have to agree to close-out netting when they sign the contract in $t = 0$ and they cannot change this aspect later.[15] First, in Section 4 I will assume that it is not possible to include close-out netting in the derivative contract. Later, in Section 5 I consider the model when close-out netting can be agreed upon.

Besides varying the model in terms of the possibility to apply close-out netting, each case is analyzed once when the firm can commit to the derivative and once when commitment is not possible. In the case of commitment, the creditor can set the contractual repayment as a function of the firm's future derivative position. Otherwise, I assume that the creditor always correctly anticipates the equilibrium and, hence, breaks even.

2.4 Timeline

First, the firm enters into a debt contract with the creditor. Afterwards, she has access to derivatives markets in order to hedge the business risk with a counterparty and subsequently the firm undertakes the initial investment. At date 1, the first cash flow of the project realizes. Then, the derivative contract

14 Note that this transfer can only be negative due to the resource constraint.

15 This assumption reflects the practice of the so-called "master agreement". When underwriting a derivative contract, counterparties can sign a "master agreement" which determines details of the underlying derivative, e.g. close-out netting. For more information on the "ISDA Master Agreement" which is often used as a blueprint for master agreements, see International Swaps and Derivatives Association (2019).

is settled. If the firm fails to repay the creditor in $t = 1$ after the derivative has been settled, the creditor seizes the firm and terminates all operations. Otherwise, the firm continues until the last period. Figure 2 illustrates the course of events of the model.

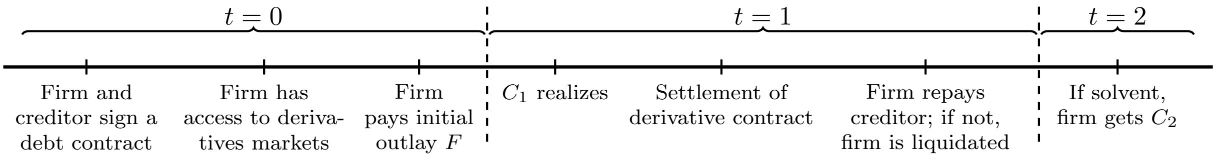

Figure 2: Timeline of the model setup.

The following remark shall simplify the understanding of the upcoming results.

Remark 1. In the following sections the firm's private profits and social surplus will coincide and sometimes I will use these terms interchangeably. Because the counterparty breaks even and the creditor can either react to the firm's derivatives portfolio or correctly anticipates the action of the firm, the firm's profits will always be equivalent to the social surplus. This implies that I do not allow for speculative behaviour of the firm. When the firm cannot commit to the derivative, she will always act according to common interest.

3 Baseline: No derivatives

As in Bolton and Oehmke (2015), I first introduce a solution of the model when the firm lacks access to derivatives. The equilibrium with no derivatives corresponds exactly to the solution of Bolton and Oehmke (2015) and serves as a benchmark for future results.

The firm needs to borrow the initial investment F from the creditor in return for a payment of R at date 1. Because the firm cannot pledge future cash flows except the low ones, she faces an incentive problem at date 1. Even if the high cash flow is realized, it might be more profitable to not repay the creditor and divert the assets. Because I have assumed that the creditor will not renegotiate, the firm extracts only $C_1^H - C_1^L$ from a strategic default. More specifically, the firm chooses to divert the cash flows in the high state whenever

$$C_1^H + C_2 - R < C_1^H - C_1^L \tag{3}$$

$$\Leftrightarrow C_2 + C_1^L < R. \tag{4}$$

From (4) I can infer that the last-period cash flow C_2 must be always greater than or equal to $R - C_1^L$ to incentivize the firm to continue until the last period. Hence, I assume in the following that this holds always true, i.e. C_2 is always large enough.

Given that the future cash flow is high enough and the firm has no access to insurance, the firm cannot serve the creditor after C_1^L. Therefore, the lender sets R according to his break-even constraint,

$$\theta R + (1-\theta)C_1^L = F, \tag{5}$$

and will demand a premium because of the additional risk. Rearranging the break-even condition yields

$$R = \frac{F - (1-\theta)C_1^L}{\theta}. \tag{6}$$

The expression (6) fully desribes the equilibrium outcome. The general features of the equilibrium with no derivatives markets are summarized in Proposition 1.

Proposition 1 (No derivatives). *Given no access to derivatives markets and assuming that C_2 is large enough, in equilibrium the creditor will demand a face value of debt equal to $R = \frac{F-(1-\theta)C_1^L}{\theta}$. The firm will always default in the low cash flow state and the social surplus, which is equal to the profit of the firm, amounts to $\theta(C_1^H + C_2) + (1-\theta)C_1^L - F$.*

Proof. See above. □

4 Basic derivative contract

This section analyzes the variation of the model setup of Bolton and Oehmke (2015) when the derivative contract does not include close-out netting. Not surprisingly, the modified model will show results similar to the ones found in Bolton and Oehmke (2015) when basis and cash flow risk are identical. At first, I discuss the contractual situation when the firm can commit to the derivative contract and, as a second step, when such commitment is not possible.

4.1 Commitment

When the firm is able to commit to the contract, the creditor is able to react. Therefore, the firm will choose the optimal contract from an ex-ante point of view because the creditor will set the contractual repayment as a function of the derivative.

Under certain conditions, the firm will use the derivative contract only as a hedge against the cash flow risk. Compared to the situation without insurance, the firm will continue operations in the low cash flow state - given that the counterparty is able to pay. Unlike in Bolton and Oehmke (2015) the firm cannot completely eliminate the risk of default even though the derivative contract directly depends on the cash flows. The firm's default is entirely driven by the the exogenous default probability of the counterparty, i.e. the firm is exposed to counterparty risk. If the counterparty risk is not too large, derivatives will increase social surplus.

In order to show this formally, consider a derivative contract with $\tau^B(C_1^L) \geq R - C_1^L$. On the one hand, a transfer of size $R - C_1^L$ or greater in the low cash flow state ensures that the firm can repay the creditor and continue its operations until the last period. But the firm receives the transfer $\tau^B(C_1^L)$ from the counterparty only when the latter stays solvent; otherwise, the counterparty cannot honor its obligation from the derivative. Because the counterparty defaults with probability $1-\pi$ in $t=1$, the net gain of $\tau^B(C_1^L) \geq R - C_1^L$ amounts to $(1-\theta)\pi C_2$. On the other hand, I assumed that due to regulatory capital requirements the counterparty bears a per-unit cost δ which she passes on to the firm. Hence, the marginal utility of a hedging position is discontinuous and the firm will choose a hedging position just sufficient to insure the risk of a low outcome C_1^L. Any further increase of $\tau^B(C_1^L)$ will only increase the price of the derivative contract because of the capital requirements and does not yield additional benefit.

The counterparty breaks even when the expected transfers to the counterparty equal the expected transfers to the firm plus the deadweight cost of hedging. Given $\tau^B(C_1^L) = R - C_1^L$, the counterparty must pay the firm after C_1^L but only when the counterparty stays solvent which happens with probability $(1-\theta)\pi$. In return, the counterparty receives a transfer $\tau^S(C_1^H)$ after C_1^H. Because the counterparty always values incoming transfers, she receives $\tau^S(C_1^H)$ with probability θ. Additionally, the counterparty must be compensated for the capital requirements associated with the deriviative contract. Hence, the break-even condition takes the form

$$\underbrace{\theta\tau^S(C_1^H)}_{=\text{payment to counterparty}} - \underbrace{(1-\theta)\pi(R-C_1^L)}_{=\text{hedging position of firm}} - \underbrace{\delta(R-C_1^L)}_{=\text{deadweight cost}} = 0. \tag{7}$$

Rearranging this term yields

$$\tau^B(C_1^H) = -\frac{[(1-\theta)\pi + \delta](R-C_1^L)}{\theta}. \tag{8}$$

Compared to the situation without derivatives, the firm will now be able to repay the creditor not only after C_1^H but also after C_1^L when the counterparty stays solvent. In total, the firm will honor her contractual obligation from the debt contract with probability $\theta + (1-\theta)\pi$. If the counterparty defaults after C_1^L, the firm cannot repay the creditor who will receive the liquidation value C_1^L. Note that the seniority of derivatives effectively plays no role here because the derivative contract is a (net) claim for the firm in the low cash flow state. Hence, the creditor will ask for a contractual repayment R which satisfies the break-even condition

$$\underbrace{[\theta + (1-\theta)\pi]R}_{=\text{firm stays solvent}} + \underbrace{(1-\theta)(1-\pi)C_1^L}_{=\text{firm defaults}} = F, \tag{9}$$

$$\Leftrightarrow R = \frac{F - (1-\theta)(1-\pi)C_1^L}{\theta + (1-\theta)\pi}. \tag{10}$$

Because of the counterparty risk the firm faces in the derivative contract, also the debt contract is not risk-free. Yet, the contractual repayment is still lower than in the case of no insurance (except if the counterparty is always defaulting, i.e. $\pi = 0$).

Given that the firm can commit to the size of the derivative in the debt contract, the equilibrium is characterized by $\tau^B(C_1^L) = R - C_1^L$, (8) and (10). This equilibrium leads to the following proposition.

Proposition 2 (Welfare properties under commitment). *Assume that the firm can commit to her derivative position in the debt contract. Then, in equilibrium the firm's profits and the overall social surplus are given by*

$$\theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta \frac{F - C_1^L}{\theta + (1-\theta)\pi}. \tag{11}$$

Compared to the situation without derivatives, these increase social surplus if

$$(1-\theta)\pi C_2 - \delta \frac{F - C_1^L}{\theta + (1-\theta)\pi} > 0. \tag{12}$$

Proof. See Appendix A1. □

Proposition 2 shows that derivatives are only useful if either the continuation value C_2 is sufficiently large, the per-unit cost of derivatives δ are not too high or the counterparty is relatively stable. In the extreme case of π equal to 1, the last condition is irrelevant like in Bolton and Oehmke (2015). For too low π, however, derivatives are individually and socially not rationale. An increase in counterparty risk has two effects: the expected gain of a derivative decreases because it is less likely that the counterparty will be able to pay after C_1^L, which, in turn, will increase the cost of the debt contract due to the higher risk the creditor has to bear. The increased cost of debt forces the firm to demand a larger amount of insurance $\tau^B(C_1^L)$ which causes a greater deadweight cost. At some point, the counterparty risk is so large that not insuring C_1^L is preferable.

4.2 No commitment

Now, I consider the situation when the firm cannot commit to the derivative contract beforehand. Covenants in debt contracts which restrict a firm's activities on derivatives markets are not common. Moreover, they are difficult to enforce especially for financial institutions because of the opaqueness of their balance sheets and business operations.[16]

With no commitment, the firm will set the derivative contract from an ex-post perspective because the creditor's repayment will not react to the contract. As long as C_2 is large enough, the transfer in the low cash flow state will be set at least to $R - C_1^L$ such that the firm continues business until the last period - if the counterparty is able to make the transfer. However, the firm will not increase $\tau^B(C_1^L)$ any further. In order to see this formally, consider the maximisation problem of the firm. The firm maximizes her profits with respect to $\tau^B(C_1^L)$ given that $\tau^B(C_1^L) \geq R - C_1^L$ and the break-even

16 For further information, see Brunnermeier and Oehmke (2013).

condition of the counterparty, i.e.

$$\begin{aligned} \max_{\tau^B(C_1^L)\geq R-C_1^L} \quad & \theta(C_1^H - R + \tau^B(C_1^H) + C_2) + (1-\theta)\pi(C_1^L - R + \tau^B(C_1^L) + C_2), \\ \text{s.t.} \quad & -\theta\tau^B(C_1^H) - (1-\theta)\pi\tau^B(C_1^L) - \delta\tau^B(C_1^L) = 0. \end{aligned} \tag{13}$$

When plugging the constraint into the objective function, the first derivative of the maximization problem,

$$-[(1-\theta)\pi + \delta] + (1-\theta)\pi < 0, \tag{14}$$

shows that excessively large hedging positions are not optimal. Because I assumed that the derivative's underlying is perfectly correlated with the cash flows of the firm's investment project, the firm has to bear the full cost of increasing $\tau^B(C_1^L)$. More specifically, if the firm wants to receive more after C_1^L, the counterparty will demand an additional amount after C_1^H in order to set off the increase $\tau^B(C_1^L)$ in expectation. Moreover, the overall deadweight cost induced by capital requirements increase as well. Therefore, it is not rational for the firm to increase $\tau^B(C_1^L)$ beyond $R - C_1^L$. In sum, the contractual equilibrium and its outcome are exactly the same as in the case of commitment (when the creditor correctly anticipates the equlibrium).

This result is the same as in Bolton and Oehmke (2015) who show the equivalence of junior and senior derivatives in the case of a perfectly correlated basis and cash flows. The additional default risk stemming from the counterparty risk has no effect and the firm will still choose the socially efficient hedging position - which might be zero under certain conditions. To sum up this section, Proposition 3 states the equivalence of the outcomes under commitment and no commitment given that contracts can only be defined on the cash flows.

Proposition 3 (Welfare properties under no commitment)**.** *Assume that the firm cannot commit to her derivative position in the debt contract. Then, the firm's profits and social surplus are the same as in the case of commitment, i.e.*

$$\theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta\frac{F - C_1^L}{\theta + (1-\theta)\pi}, \tag{15}$$

and derivatives increase social surplus when

$$(1-\theta)\pi C_2 - \delta\frac{F - C_1^L}{\theta + (1-\theta)\pi} > 0. \tag{16}$$

Proof. See Appendix A2. □

5 Contracts with netting

In this section, close-out netting can be included in the derivative contract. As before, at first, I consider the situation when the firm can commit to the derivative contract before examining the effect of no commitment afterwards.

5.1 Commitment

A contract between firm and counterparty now encompasses five transfers instead of only two. After C_1^H either $\tau^B(C_1^H, 0, 0)$ or $\tau^B(C_1^H, 0, 1)$ are exchanged. While the former transfer can be positive or negative, the latter can only be negative, i.e. the counterparty can only receive something after her own default. A default of the counterparty after C_1^L triggers the default of the firm since the latter has no chance of making the contractual repayment R and only a transfer $\tau^B(C_1^L, 1, 1) \leq 0$ can occur. A priori, $\tau^B(C_1^L, 0, 0)$ and $\tau^B(C_1^L, 1, 0)$ can be due after C_1^L if the counterparty stays solvent. If $\tau^B(C_1^L, 0, 0)$ is not sufficient to repay the creditor, the firm defaults after C_1^L despite the counterparty being solvent. For this case, the derivative contract specifies $\tau^B(C_1^L, 1, 0)$. However, I have assumed that C_2 is large enough to incentivize the firm to repay the creditor. Hence, $\tau^B(C_1^L, 0, 0)$ will be set always greater than or equal to $R - C_1^L$. This implies that $\tau^B(C_1^L, 1, 0)$ never comes into play and subsequently it can be set arbitrarily. For brevity, I leave it out in the following.

From the remaining four transfers, namely $\tau^B(C_1^H, 0, 0), \tau^B(C_1^H, 0, 1), \tau^B(C_1^L, 0, 0)$ and $\tau^B(C_1^L, 0, 1)$, I can see that the indicator variable D_F does not add substantial information to the contract. Because the firm only defaults after a low cash flow and the counterparty's default, whereas she stays solvent in all other states, a contract which depends only on (C_1, D_C) already describes the whole state space. Therefore, I can simplify notation and make contracts dependent on

$$(C_1, D_C) \in \{C_1^L, C_1^H\} \times \{0, 1\}. \tag{17}$$

The situation with commitment actually features a continuum of equilibria because of the assumption that the counterparty still values incoming transfers even after a default. First, note that the firm cannot shift resources from $(C_1^H, 1)$ to $(C_1^H, 0)$ and increase her profits. If the other transfers stay constant, an increase in $\tau^B(C_1^H, 0)$ would be, in expectation, (more than) offset with a decrease in $\tau^B(C_1^H, 1)$ because the counterparty needs to break even. Therefore, one can view $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^H, 1)$ as substitutes. In equilibrium, the firm will be indifferent between any set of feasible contracts as long as $\tau^B(C_1^L, 0)$ hedges the business risk in the low cash flow state, $\tau^B(C_1^L, 1)$ is set to zero and $\tau^B(C_1^H, 0)$ does not exceed $R - C_1^L$.

In order to show this formally, I must solve the model by backward induction. Hence, I first consider the derivative contract. As before, the firm at least wants to hedge the business risk in the low cash flow state which is only possible if the firm stays solvent. The transfer $\tau^B(C_1^H, 0)$ will either be smaller than the hedge or it will be of the same size in order to economize on the deadweight costs. To be clearer on the latter part of this statement, suppose that $\tau^B(C_1^H, 0)$ is greater than $\tau^B(C_1^L, 0) \geq R - C_1^L$. Then, $\tau^B(C_1^H, 0)$ constitutes the maximum exposure of the counterparty and, hence, determines the cost of capital which the counterparty passes on to the firm. In this situation, the firm could increase her profits by lowering $\tau^B(C_1^H, 0)$ and increasing $\tau^B(C_1^L, 0)$ overproportional. Because a decrease of $\tau^B(C_1^H, 0)$ not only relaxes the break-even condition but also reduces the total derivative cost, the firm can increase $\tau^B(C_1^L, 0)$ more than $\tau^B(C_1^H, 0)$ is reduced. This logic implies that in the optimum, I will have

$$\tau^B(C_1^H, 0) = \tau^B(C_1^L, 0), \tag{18}$$

if $\tau^B(C_1^H, 0)$ is greater than or equal to $R - C_1^L$. Therefore, I can write the counterparty's break-even condition as

$$\theta[\pi\tau^S(C_1^H, 0) + (1-\pi)\tau^S(C_1^H, 1)] + (1-\theta)[\pi\tau^S(C_1^L, 0) + (1-\pi)\tau^S(C_1^L, 1)] = \delta\tau^B(C_1^L, 0), \tag{19}$$

with which I can derive

$$\tau^B(C_1^H, 0) = -\frac{[(1-\theta)\pi + \delta]\tau^B(C_1^L, 0) + \theta(1-\pi)\tau^B(C_1^H, 1) + (1-\theta)(1-\pi)\tau^B(C_1^L, 1)}{\theta\pi}. \tag{20}$$

The firm could pay the counterparty something in the case when both default.[17] The cost, however, would be borne by the creditor who only has junior claims. When the creditor is able to react, her break-even condition,

$$\theta R + (1-\theta)[\pi R + (1-\pi)(C_1^L + \tau^B(C_1^L, 1))] = F, \tag{21}$$

is equivalent to

$$R = \frac{F - (1-\pi)(1-\theta)(C_1^L + \tau^B(C_1^L, 1))}{\theta + (1-\theta)\pi}. \tag{22}$$

Remember that $\tau^B(C_1^L, 1)$ must be negative because the counterparty defaults. I can use the formulas (20) and (22) to state Lemma 1.

Lemma 1 (No risk-shifting)**.** *Assume the firm can commit to the derivative contract and the derivative contract is defined on all states* (C_1, D_C)*. Then, I have in equilibrium*

$$\tau^B(C_1^L, 1) = 0. \tag{23}$$

17 Note that this is also possible when the counterparty defaults after C_1^H. However, this has not the risk-shifting effect as in the case when both default because I have assumed that C_1^H is high enough and the firm prefers to remain solvent even more after the high cash flow state.

Proof. See Appendix A3. □

Under commitment, the refined derivative contract still leads qualitatively to the same equilibrium outcome I observed in the previous section - the transfers in the high cash flow states only induce multiple contractual equilibria which all feature no risk-shifting. The reason is that the creditor can react to any attempt of the firm to expropriate her in the state where both firm and counterparty default. Any collateral the firm receives less in this state will increase the contractual repayment and, therefore, the hedging position the firm needs. But a larger hedging position leads to an increase in costs for the counterparty who, in turn, passes these on to the firm. All in all, the firm has no incentive to actually shift risk from the counterparty to the creditor. Note further that the priority treatment of derivatives becomes effectively irrelevant in the equilibrium (as before). Only the creditor has claims against the defaulting firm and gets to seize the low cash flow. The equilibria of this subsection are summarized in Lemma 2.

Lemma 2 (Equilibria with commitment)**.** *Assume the firm can commit to the derivative contract and the derivative contract is defined on all states* (C_1, D_C)*. Then, the set of all equilibria consists of the contractual repayment*

$$R = \frac{F - (1-\pi)(1-\theta)C_1^L}{\theta + (1-\theta)\pi} \tag{24}$$

and

$$\tau^B(C_1^H, 0) \in \Big[-\frac{\delta + (1-\theta)\pi}{\theta + (1-\theta)\pi}\frac{F - C_1^L}{\theta\pi}, \frac{F - C_1^L}{\theta + (1-\theta)\pi}\Big], \tag{25}$$

$$\tau^B(C_1^H, 1) = -\frac{\theta\pi}{\theta(1-\pi)}\tau^B(C_1^H, 0) - \frac{\delta + (1-\theta)\pi}{\theta + (1-\theta)\pi}\frac{F - C_1^L}{(1-\pi)\theta}, \tag{26}$$

$$\tau^B(C_1^L, 0) = \frac{F - C_1^L}{\theta + (1-\theta)\pi}, \tag{27}$$

$$\tau^B(C_1^L, 1) = 0. \tag{28}$$

Proof. See Appendix A4. □

At last, the firm will choose the socially efficient outcome because, if possible, the firm continues operations until $t = 2$ and the deadweight cost are reduced to a minimum. Despite the difference compared to the previous section that neither firm nor counterparty have any exposure from the derivative contract if both default, the cost of capital requirements stay the same.[18] Proposition 4 concludes this subsection.

18 This result might differ if one assumes that the per-unit derivative cost δ are risk-weighted, i.e. reflect the exposure stemming from the derivative contract in a fundamental way.

Proposition 4 (Welfare properties with netting under commitment)**.** *Assume that the firm can commit to her derivative position in the debt contract and the derivative contract is defined on all states* (C_1, D_C)*. Then, the equilibrium outcome is socially equivalent to the case of the basic derivative contract.*

Proof. See Appendix A5. □

5.2 No commitment

Now, I consider the situation when commitment is not possible. Analogously to the section with the basic derivative contract, the firm maximizes her profits from an ex-post perspective because the creditor cannot react.

Without commitment, I will find that the well-known risk-shifting problem arises because the firm uses the option to refine the contract as a measure to expropriate the creditor in case of default. Moreover, there is the possibility that both $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^L, 0)$ actually exceed $R - C_1^L$ if the shortfall in the firm's balance sheet is not too large, i.e C_1^L is relatively close to R. Therefore, I need to make a case distinction. Unlike in the previous section, however, I restrict our attention to equilibria where $\tau^B(C_1^H, 1)$ is equal to zero. Since this transfer cannot be used as a risk-shifting device, this assumption only simplifies matters without changing the qualitative results.

At first, suppose that $\tau^B(C_1^H, 0)$ will be lower than $R - C_1^L$. The counterparty's break-even condition will be equivalent to the case of commitment and yield expression (20) for $\tau^B(C_1^H, 0)$. Given that the hedging position is large enough to insure the low cash flow if the counterparty stays solvent, the firm maximizes

$$\max_{\tau^B(C_1^L,0) \geq R - C_1^L} \quad \theta(C_1^H - R + \tau^B(C_1^H, 0) + C_2) + (1-\theta)\pi(C_1^L - R + \tau^B(C_1^L, 0) + C_2), \tag{29}$$

such that (20) holds. The first derivative with respect to $\tau^B(C_1^L, 1)$ is

$$-(1-\theta)(1-\pi) < 0. \tag{30}$$

This implies that setting $\tau^B(C_1^L, 1)$ as low as possible is optimal because it allows the firm to negotiate better terms with the counterparty. The firm might utilize the individual gains from the risk-shifiting in two different ways. At first, the firm will shift the additional resources to state $(C_1^H, 0)$ until the corresponding transfer is as large as the hedge after C_1^L because, up to the point where $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^L, 0)$ are equal, the marginal gain from increasing the former is higher than the marginal gain from increasing the latter. This is due to the cost of capital which the counterparty passes to the firm. Indeed, an increase in $\tau^B(C_1^L, 0)$ caused by the risk-shifting leads to higher profits, but only at the rate of $(1-\delta)$ per-unit. An increase in $\tau^B(C_1^H, 0)$ does not result in higher capital requirements and, hence, is more profitable. If, however, $\tau^B(C_1^H, 0)$ reaches $R - C_1^L$, e.g. because the risk-transfer is very large, it is rational to increase both transfers simultaneously as has been shown in the case of commitment.

The complete contractual equilibrium of this subsection is summarized in Lemma 3.

Lemma 3 (Equilibrium with netting under no commitment)**.** *Assume the firm cannot commit to the derivative contract and the derivative contract is defined on all states* (C_1, D_C)*. Moreover, assume that counterparty and firm agree to set* $\tau^B(C_1^H, 1)$ *to zero. If*

$$\frac{1+\delta-(1-\pi)\theta}{\pi+\delta} < \frac{R}{C_1^L} \tag{31}$$

holds true, the optimal contract consists of,

$$\begin{aligned} \tau^B(C_1^H, 0) &= -\frac{[(1-\theta)\pi+\delta](R-C_1^L)-(1-\theta)(1-\pi)C_1^L}{\theta\pi}, \\ \tau^B(C_1^L, 0) &= R - C_1^L, \\ \tau^B(C_1^L, 1) &= -C_1^L. \end{aligned} \tag{32}$$

If (31) *does not hold, the optimal contract takes the following form,*

$$\begin{aligned} \tau^B(C_1^H, 0) = \tau^B(C_1^L, 0) &= \frac{(1-\theta)(1-\pi)C_1^L}{\pi+\delta}, \\ \tau^B(C_1^L, 1) &= -C_1^L. \end{aligned} \tag{33}$$

Proof. See Appendix A6. □

The main result of this section is that, in equilibrium, the firm will shift some of the risk from the counterparty to the creditor in order to get better terms from the counterparty.[19] This risk-shifting is a well-known feature which arises in a number of different setups.[20] All of the models feature the problem of limited commitment. The problem of risk-shifting in our setup is also caused by the seniority of derivatives.[21] The firm can only promise the counterparty C_1^L in the case both default, because the counterparty enjoys a higher priority than the creditor. Put differently, the creditor can be diluted. If derivatives were junior, this problem would not arise and the firm would choose a more efficient contract (from the viewpoint of the creditor). However, it must be emphasized that, in this model setup, the seniority of derivatives is a necessary but not a sufficient condition for the risk-transfer. In Section 5, when the derivative was defined on the cash flows, the risk-shifting did not arise despite the senior treatment of derivatives. The risk-shifting has become possible because of close-out netting.

19 For the sake of completeness, it is important to emphasize that this holds also true for equilibria where $\tau^B(C_1^H, 1)$ is different from zero.

20 In Brunnermeier and Oehmke (2013) the limited liability paired with multiple possible maturities is the root of the problem, while in Bolton and Oehmke (2015) the basis, which is not perfectly correlated with the cash flows of the firm, induces the risk-shifting. In Donaldson and Piacentino (2019) banks even willingly take on junior debt claims to have the option to dilute themselves later.

21 As stated above, the seniority of derivatives is simplifying. However, it plasticises the difference to the situation when derivatives are junior to debt and no risk-shifting happens. Moreover, the risk-shifting problem arises even when assuming that only a fraction of the counterparty's claims is senior in order to account for the legal advantages of derivatives.

Although there are many necessary conditions for the risk-shifting, the model shows that risk-shifting might arise when the ability to define contracts varies across creditors. Then, a firm might use more detailed contracts as a measure to shift risk on the creditors whose contracts encompass less state-contingent details. As I have interpreted close-out netting, in our model the derivative contract becomes a complete contract due to exogenous factors, i.e. legal rules.[22]

From a welfare-perspective, the risk-shifting is inefficient because it lowers private profits and destroys social surplus. If the scope of risk-shifting is greater, e.g. because C_1^L is almost as large as F, the loss in social surplus is even more dramatic. The intuition behind this result is simple. Because the creditor will anticipate that the firm will securitise C_1^L to the counterparty if both default, the creditor demands a contractual repayment of

$$\theta R + (1-\theta)[\pi R + (1-\pi)\cdot 0] = F,$$
$$\Leftrightarrow R = \frac{F}{\theta + (1-\theta)\pi}. \tag{34}$$

This contractual repayment is greater than in any of the previous cases (with derivatives). Therefore, the firm needs a larger hedging position $\tau^B(C_1^L, 0)$ which leads to a greater deadweight cost. This effect is even more pronounced when the risk-shifting allows the counterparty to increase $\tau^B(C_1^L, 0)$ (and $\tau^B(C_1^H, 0)$) above $R - C_1^L$. The last result of this section is summarized by Proposition 5.

Proposition 5 (Welfare properties with netting under no commitment)**.** *Assume that the firm cannot commit to her derivative position in the debt contract and the derivative contract is defined on all states* (C_1, D_C)*. If* (31) *holds true, derivatives increase the firm's profits and social surplus when*

$$(1-\theta)\pi C_2 - \delta\left(\frac{F}{\theta + (1-\theta)\pi} - C_1^L\right) > 0, \tag{35}$$

while, if (31) *does not hold true, derivatives increase the firm's profits and social surplus when*

$$(1-\theta)\pi C_2 - \delta\frac{(1-\theta)(1-\pi)}{\pi + \delta}C_1^L > 0. \tag{36}$$

In both cases, the social surplus is strictly less than in the case of commitment or only basic derivative contracts.

Proof. See Appendix A7. □

A crucial assumption, which has not yet been discussed, is the counterparty's valuation of incoming transfers in the case of its default. One of the main reasons that risk-shifting can actually take place is the assumption that the counterparty still values incoming transfers even if it defaulted. The

22 Another possibility is that some creditors receive more information than others and also have the ability to write contracts dependent on this additional information. Such setups arise in financial markets where some agents are better informed than others.

counterparty does not make a difference between one dollar she received when she is solvent and one dollar she received when she defaulted. This assumption seems artificial given that common insolvency law, which is applied in this model to the firm, would lead to the counterparty's creditors taking over the counterparty when her assets are deployed. I did not assume the existence of such creditors, yet it is instructive to rethink the previous results in the case of an insolvency process for the counterparty. To that end, assume that the counterparty's default implies that there is an imbalance in claims and liabilities such that the counterparty must declare default and her creditors get all incoming transfers. If the counterparty, like the firm, cares only for the residual profits, any transfer in the low cash flow state where the imbalance occurs is not valued. Therefore, I have that even the refinement of the derivative contract eventually will not induce risk-shifting, i.e. when the counterparty's shortfall on the asset side is very large. If the transfer, however, is sufficient to "push" the counterparty into solvency such that it actually assigns a positive value to this transfer, I have a similar equilibrium outcome as described in Lemma 3 only with slightly worse terms for the firm in the derivative contract.

To sum up, changing the assumption on the insolvency process of the counterparty in the way described above would have the same effect as assuming that derivatives are junior to debt. It would take away the possibility of dilution for a lack of valuation from the counterparty.[23]

5.3 Discussion

After having derived the implications of refined derivative contracts, which I have presented as a way to model netting, the question arises how these can be interpreted in a practical context. Put differently, the question is whether equivalent situations arise in practice, namely, by using netting arrangements (or not).

In the case of commitment, the solution to the model shows that, in reality, close-out netting agreements could lead to such situations. On the one hand, after C_1^L, the firm will receive the hedge if the counterparty stays solvent while there will be no transfer if the counterparty defaults. This is as if the firm has a derivatives portfolio which constitutes a net claim in the low cash flow states. Close-out netting will erase any liabilities to the counterparty.[24] On the other hand, the equilibrium features transfers to the counterparty after C_1^L, i.e. $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^H, 1)$ are both negative. Hence, the derivative is a net liability to the counterparty and, therefore, the firm has to pay the counterparty also after the latter's default.

In the case of no commitment, there are states where netting will not appear and the firm rather keeps a gross position, namely in the state where the counterparty's default leads to the firm's default. The firm utilizes the transfer in this state to expropriate the creditor. In reality, such a transfer is only possible, if firm and counterparty do not apply close-out netting. Then, any (unsecured) claims of the

23 Note that a similar question does not arise for the firm by construction.

24 The fact that the counterparty's default triggers also the firm's default is of no importance.

firm decay while she still has to serve the counterparty. With a netting arrangement, a similar transfer can only arise if the firm has a net liability in $(C_1^L, 1)$. However, because the other transfer after C_1^L, i.e. the insurance, is a claim for the firm, this seems unrealistic.

6 Extensions

Even though a lot of derivatives can significantly help firms to hedge risks inherent in their business activity because they are very sensitive to the underlying risk, e.g. an airline who wants to hedge their cost for fuel with commodity futures, other derivatives still have the risk that the underlying is not completely aligned with the operations of the firm. Therefore, it is necessary to discuss the implications of basis risk in this model setup, i.e. the risk caused by an imperfect correlation between the cash flows and the underlying. I take the definition of Bolton and Oehmke (2015) where the derivative is defined on "a verifiable random variable $Z \in \{Z^H, Z^L\}$" (Bolton and Oehmke (2015), p. 2360), hereafter called basis, with the property that

$$P(Z = Z^L | C_1 = C_1^L) = \gamma \in [0, 1) \tag{37}$$

and

$$P(Z = Z^L) = 1 - \theta. \tag{38}$$

In the previous sections, I have assumed that $\gamma = 1$ such that the basis was low whenever the cash flow was low. Now, with $\gamma < 1$ it can happen that despite a low cash flow the basis is in the high state.[25]

A basic derivative contract is only defined on the basis and, hence, specifies payments $\tau^B(Z^H)$ and $\tau^B(Z^L)$.[26] With basis risk, the firm faces now an additional problem. Because the counterparty needs to be compensated for providing the insurance, the firm has to decide after which basis state she wants to stay solvent. Put differently, after C_1^L the firm defaults if either the counterparty defaults or the derivative becomes a liability due to the "wrong" basis state. Moreover, the hedging position constraints the counterparty's break-even condition not only after C_1^L but also after C_1^H. This stems from the basis risk of the derivative and leads to further inefficiencies. In general, our model with basis risk and derivative contracts only defined on the basis confirms the results of Bolton and Oehmke (2015) regarding the behaviour of the firm under commitment and no commitment. The main differences are the following.

25 Note that γ close to zero implies a negative correlation between cash flow and basis - although not perfectly negative.

26 Note that $\tau^B(Z^H)$ corresponds to the (reverse of the) premium x of the derivative contract in Bolton and Oehmke (2015) whereas $\tau^B(Z^L)$ is the same as the notional X.

First, the default possibility of the counterparty decreases the private and social incentives to use derivatives. This is because the expected value of continuing operations decreases in π and a lower π leads to greater cost of debt and, hence, a greater deadweight cost related to the derivative.

Second, the range of values for which hedging is actually efficient under commitment is greater than in Bolton and Oehmke (2015).[27] Unlike Bolton and Oehmke (2015) I have placed no restrictions on the transfers $\tau^B(Z^H)$ and $\tau^B(Z^L)$ - except the resource constraints - and, therefore, the firm can also choose $\tau^B(Z^H)$ (instead of $\tau^B(Z^L)$) as a hedging position. The firm will use $\tau^B(Z^H)$ as insurance if γ is close to 0 as this implies that Z^H is more likely after C_1^L.[28] All in all, there will be thresholds $\underline{\gamma}$ and $\bar{\gamma}$ such that derivatives are privately rational and efficient if γ is either below $\underline{\gamma}$ or above $\bar{\gamma}$.[29] If the set $(\underline{\gamma}, \bar{\gamma})$ is non-empty, derivatives are inefficient for some parameter values of γ. This is more likely if both the default probability of the firm, $1-\pi$, and the per-unit deadweight cost of derivatives, δ, are higher. Then, the gains of the derivative are low because the derivative is too unsecure as a hedge, i.e. the correlation between cash flows and basis is very weak, and the cost of the derivative are high.

With netting, the derivative contract is extended to the state space (Z, D_F, D_C). More specifically, because the derivative contract is now defined on a basis which can be either high or low in both cash flow states, there are more transfers which can be chosen, namely

$$(Z, D_F, D_C) \in \{Z^H, Z^L\} \times \{0,1\} \times \{0,1\} \tag{39}$$

where, similar to before, $\{D_F = 1\}$ indicates the default of the firm and $\{D_C = 1\}$ indicates the default of the counterparty. Essentially, the transfers $\tau^B(Z^H, 1, 0)$ and $\tau^B(Z^L, 1, 0)$ are less relevant and can be set arbitrarily. Both of these transfers would occur only after C_1^L when the firm defaulted despite the counterparty being able to serve the derivative contract. Because I assume that C_2 is large enough, the firm wants to continue until the last period and, hence, the firm will use $\tau^B(Z^H, 0, 0)$ and $\tau^B(Z^L, 0, 0)$ to insure the business risk, i.e. both of these transfers are set sufficiently high such that the firm never defaults when the counterparty stays solvent. Moreover, note that also in the case of a derivative contract with netting, the derivative might constitute a liability while the firm receives only C_1^L from the investment project. However, with netting the firm has the ability to consciously plan such situations whereas before these happened only due to "bad luck".

The remaining six transfers can be simplified further. Without loss of generality, two pairs of these transfers will be equal to each other in any equilibrium. First, the firm is indifferent with regard to any allocation between the states where both counterparty and firm default because the circumstances are

27 This statement holds also true if I would have assumed that derivatives are junior to debt in the insolvency process. Moreover, as has been shown by Bolton and Oehmke (2015), the social surplus (and the incentive to hedge) are strictly greater with junior derivatives.

28 Because a γ close to 0 does not imply that the basis and the cash flows are almost perfectly negatively correlated, the transfer $\tau^B(Z^H)$ will still occur after C_1^H. Therefore, the assumption that C_1^H is sufficiently large to repay the counterparty after C_1^H becomes more stringent for low values of γ than for high values.

29 Unfortunately, I was not yet able to derive a nice closed-form solution for the thresholds $\underline{\gamma}$ and $\bar{\gamma}$.

the same except that the basis is either high or low. In both cases the firm can only make payments to the counterparty, who attaches the same value to both and the transfers will have the same effect with respect to the counterparty's break-even condition and, under commitment, the creditor's reaction. The exact same argument holds true for the states where only the counterparty defaults. Therefore, the maximization problem of the firm breaks down to essentially choosing only four transfers.

Depending on whether commitment is possible or not, the transfers $\tau^B(Z^H, 1, 1)$ and $\tau^B(Z^L, 1, 1)$ will be set as in the previous section. If the firm can credibly commit, the creditor's reaction to the derivative contract forces the firm to set both transfers to zero, whereas under no commitment, the firm utilizes these transfers to dilute the creditor.[30]

As in the basic derivative contract, the main difference in the maximization problem in contrast to the case of a perfect correlation between cash flows and basis is that the firm's hedging positions, i.e. $\tau^B(Z^H, 0, 0)$ and $\tau^B(Z^L, 0, 0)$, affect the counterparty's break-even condition not only after C_1^L but also after C_1^H. For example, if the firm chooses $\tau^B(Z^L, 0, 0)$ to insure the business risk, i.e. set this transfer equal to $R - C_1^L$, the counterparty potentially has a liability after both C_1^L and C_1^H whereas before it was sure that this was a liability only after C_1^L.

Because of this difference, the firm's considerations change also with regard to the other transfers. More specifically, the transfers $\tau^B(Z^H, 0, 1)$ and $\tau^B(Z^L, 0, 1)$ gain relevance because the firm must use them to satisfy the counterparty's break-even condition in return for the hedge. Since C_2 is sufficiently large such that the firm wants to always continue after C_1^L if the counterparty stays solvent, these transfers pose the only possibility to compensate the counterparty in the case of commitment. In the case of no commitment, the firm additionally can dilute the creditor with the transfers $\tau^B(Z^H, 1, 1)$ and $\tau^B(Z^L, 1, 1)$. If the shortfall in the firm's balance sheet is large enough such that the these transfers do not suffice for the counterparty to break-even, the firm must utilize also $\tau^B(Z^H, 0, 1)$ and $\tau^B(Z^L, 0, 1)$.

The above analysis leads to the following results.

Lemma 4 (Equilibrium under commitment with basis risk)**.** *Assume the firm can commit to the derivative contract and the derivative contract is defined on all states (Z, D_F, D_C). Then, without loss of generality, in equilibrium the derivative contract consists of*

$$\tau^B(Z, 0, 0) = \frac{F - C_1^L}{\theta + (1-\theta)\pi}, \tag{40}$$

$$\tau^B(Z, 0, 1) = -\frac{\delta + \pi}{\theta(1-\pi)}\left[\frac{F - C_1^L}{\theta + (1-\theta)\pi}\right], \tag{41}$$

$$\tau^B(Z, 1, 1) = 0, \tag{42}$$

for $Z \in \{Z^H, Z^L\}$. The contractual repayment amounts to $R = \frac{F-(1-\theta)(1-\pi)C_1^L}{\theta+(1-\theta)\pi}$.

Proof. See Appendix A8. □

30 The formal proof of this result is analogous to the case of no basis risk in section 5.

Lemma 5 (Equilibrium under no commitment with basis risk)**.** *Assume the firm cannot commit to the derivative contract and the derivative contract is defined on all states* (Z, D_F, D_C)*. Then, the firm will set* $\tau^B(Z, 1, 1)$ *equal to* $-C_1^L$ *for* $Z \in \{Z^H, Z^L\}$ *in any case. Furthermore, without loss of generality, the derivative contract consists of*

$$\tau^B(Z, 0, 0) = R - C_1^L, \tag{43}$$

$$\tau^B(Z, 0, 1) = -\frac{[\delta + \pi](R - C_1^L) - (1 - \theta)(1 - \pi)C_1^L}{\theta(1 - \pi)}, \tag{44}$$

for $Z \in \{Z^H, Z^L\}$*, if condition* (31) *is satisfied. Otherwise, the equilibrium transfers are*

$$\tau^B(Z, 0, 0) = \frac{(1 - \theta)(1 - \pi)C_1^L}{\theta + \pi}, \tag{45}$$

$$\tau^B(Z, 0, 1) = 0, \tag{46}$$

for $Z \in \{Z^H, Z^L\}$.

Proof. See Appendix A9. □

All in all, the introduction of basis risk does not alter the contractual arrangements in equilibrium if netting is possible. Moreover, also in terms of efficiency nothing changes. In the case of commitment the social surplus as well as the incentives to hedge are strictly greater than in the case of no commitment. For the social surplus, however, it is important that the derivative with netting uses all variables Z, D_F and D_C this time. If the derivative would only be defined on (Z, D_C), the firm could not differentiate the default states of the counterparty. Since the counterparty causes the firm's default only after C_1^L, the indicator variable D_F makes it possible to pay the defaulted counterparty after C_1^H and not, at the same time, expropriate the creditor in case of default. Therefore, the variable D_F ensures that the firm maximizes her profits and, thereby, the maximum social surplus is reached in the case of commitment. In the case of no commitment, it prevents that the firm unnecessarily raises the transfers $\tau^B(Z^H, 0, 0)$ and $\tau^B(Z^L, 0, 0)$ even further which would increase the deadweight cost.

Furthermore, netting in the derivative contract proves to be socially beneficial as it increases the social surplus much more than the basic derivative. One the one hand, the extension of the state space allows the firm to always stay solvent after C_1^L when the counterparty stays solvent, i.e. exactly as in the case without basis risk.[31] On the other hand, the creditor acknowledges this which decreases the contractual repayment and, therefore, the hedging needs of the firm. With netting, the effect of the basis risk eventually vanishes and the social surplus reaches the same level as before. As social surplus

31 Note that the marginal cost of hedging in the different basis states is discontinuous (and depends on the possibility to commit or not). Under commitment, increasing the first transfer to the necessary hedge is more costly than increasing afterwards also the transfer for the other potential default state because the former not only takes resources from other states away but also increases the deadweight cost. Once one hedging position has been established the second one will just lead to reallocating resources.

and the profits of the firm coincide, this implies that also the firm would choose netting over the basic derivative contract.

Proposition 6 (Welfare properties with basis risk)**.** *In the presence of basis risk, the firm's profits and social surplus are greatest if commitment and close-out netting are possible.*

Proof. See Appendix A10. □

7 Conclusion

Nowadays, close-out netting has become a standard tool in derivatives markets. Not least due to the relentless efforts of the International Swaps and Derivatives Association, close-out netting is legally protected against usual insolvency requirements. While the industry argues that close-out netting contributes substantially to the stability of OTC markets in particular and financial markets in general, legal and economic scholars have expressed their doubts on this argument. I add to this discussion by modeling close-out netting theoretically and examining the reasons for its ubiquitious use. With close-out netting, derivatives counterparties are able to manage their portfolio more closely. Therefore, I have modeled close-out netting as a legal instrument which extends the state space for derivatives contracts. By applying close-out netting, derivatives contracts become complete and gain an advantage when compared to other financial contracts.

More specifically, I extend the model of Bolton and Oehmke (2015) and introduce counterparty risk and our definition of close-out netting. I find that close-out netting is at best useless if derivatives fully replicate the business risk of the firm. If debt holders lack the possibilities to monitor the firm's financing structure, the firm even utilizes close-out netting to shift risk from the counterparty to the debt holders. This is consistent with the risk-shifting problems which arise with limited commitment, e.g. in Brunnermeier and Oehmke (2013). If derivatives do not fully replicate the business risks of firms, this advantage is not only a private benefit but might increase also social surplus, i.e. close-out netting might be better than a basic derivative contract despite the risk-shifting problem.

However, our definition of close-out netting has certain shortcomings which should be addressed in future work on close-out netting. First, the current definition of close-out netting focusses almost exclusively on the netting part. Because the derivative contract only lasts for one period, there is no need to close out the contract. However, the industry stresses that the option to close out contracts is vital to a proper functioning of OTC markets since it protects non-defaulting counterparties of long-lasting insolvency procedures and further losses on the balance sheet. Secondly, our model does not feature the interaction between close-out netting and capital requirments. The time dimension of our model is too narrow in order to include this interaction. As capital requirements are in general a big point of discussion, this point must not be excluded. Future work could extend the time horizon of the model to more periods and include derivatives contracting on subsequent points in time while the need for derivatives must always be prevalent. Moreover, the solvency of the counterparty could depend on her capital. Then, both close-out netting and the interaction thereof with capital requirements could be introduced into this framework.

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Appendix

A1 Proof of Proposition 2

As the contractual equilibrium has already been calculated, it remains to deduct (11) and (12). Because the counterparty and the creditor break even, i.e. they both have zero profits, the overall social surplus corresponds exactly to the firm's profits which are given by

$$\Pi = \theta(C_1^H + C_2 - R + \tau^B(C_1^H)) + (1-\theta)\pi(C_1^L + C_2 - R + \tau^B(C_1^L)). \tag{A1}$$

Plugging the expressions for the transfers of the derivative contract and the contractual repayment into Π leads to

$$\Pi = \theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta\frac{F - C_1^L}{\theta + (1-\theta)\pi}. \tag{A2}$$

The firm is only willing to use the derivative as a hedge if the profits with hedging exceed the profits without hedging, i.e.

$$\Pi > \theta(C_1^H + C_2) + (1-\theta)C_1^L - F. \tag{A3}$$

Rearranging this inequality leads to the condition under which derivatives are privately and socially optimal.

□

A2 Proof of Proposition 3

Because the contractual equilibrium under no commitment is the same as the contractual equlibrium with commitment and the creditor correctly anticipates the firm's behaviour, the welfare properties must be the same. For a derivation, see the previous proof.

□

A3 Proof of Lemma 1

By using (20) and (22), we can infer the profit of the firm as a function of $\tau^B(C_1^L, 0)$ given that $\tau^B(C_1^L, 0) \geq R - C_1^L$, namely

$$\Pi = \theta C_1^H + [\theta + (1-\theta)\pi]C_2 + (1-\theta)C_1^L - \delta\tau^B(C_1^L, 0) - F. \tag{A4}$$

The first derivative of Π with respect to $\tau^B(C_1^L, 0)$ is negative. Hence, the firm has no incentive to increase the hedging position $\tau^B(C_1^L, 0)$ beyond $R - C_1^L$ as this causes only unnecessary deadweight cost. Note that this implies that the firm has also no incentive to set $\tau^B(C_1^H, 0)$ higher than $R - C_1^L$ because of the same reason.

We can plug $\tau^B(C_1^L, 0) = R - C_1^L$ into Π and use expression (22) for the contractual repayment which leads to

$$\Pi = \theta C_1^H + [\theta + (1-\theta)\pi]C_2 + (1-\theta)C_1^L - \delta\frac{F - C_1^L}{\theta + (1-\theta)\pi} - F + \delta\frac{\tau^B(C_1^L, 1)}{\theta + (1-\theta)\pi}. \tag{A5}$$

Since $\tau^B(C_1^L, 1)$ can only be negative and the first derivative of expression (A5) with respect to $\tau^B(C_1^L, 1)$ is positive, it follows that $\tau^B(C_1^L, 1)$ will optimally be set to zero.

□

A4 Proof of Lemma 2

From Lemma 1, we know that $\tau^B(C_1^L, 1)$ is optimally set to zero. The contractual repayment follows immediately from plugging $\tau^B(C_1^L, 1)$ into (22).

Moreover, we know that

$$\tau^B(C_1^L, 0) = R - C_1^L = \frac{F - (1-\theta)(1-\pi)C_1^L}{\theta + (1-\theta)\pi} - C_1^L = \frac{F - C_1^L}{\theta + (1-\theta)\pi}. \tag{A6}$$

At last, the ranges of values for the transfer $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^H, 1)$ follow from the two extreme cases that can emerge. The upper bound stems from the fact that $\tau^B(C_1^H, 0)$ will not exceed $R - C_1^L$. The lower bound is reached when $\tau^B(C_1^H, 1)$ is set to zero such that $\tau^B(C_1^H, 0)$ compensates the counterparty entirely for providing the hedging position.

□

A5 Proof of Proposition 4

In any of the contractual equilibria established in Lemma 2, the firm's profits amount to

$$\begin{aligned}\Pi = \theta(C_1^H + C_2 - R + \pi\tau^B(C_1^H, 0) + (1-\pi)\tau^B(C_1^H, 1)) \\ +(1-\theta)\pi(C_1^L + C_2 - R + \tau^B(C_1^L, 0)).\end{aligned} \tag{A7}$$

By plugging in the transfers of Lemma 2, we get

$$\Pi = \theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta\frac{F - C_1^L}{\theta + (1-\theta)\pi} \tag{A8}$$

which corresponds exactly to the firm's profits from the basic derivative contract. Hence, the condition for derivatives to be efficient must also be the same.

□

A6 Proof of Lemma 3

We have already shown that $\tau^B(C_1^L,1)$ will be set equal to $-C_1^L$ and we assume that $\tau^B(C_1^H,1)$ will be set equal to zero. Moreover, the firm will shift the additional resouces gained by the risk-shifting first to the state $(C_1^H,0)$, i.e. she increases $\tau^B(C_1^H,0)$. When $\tau^B(C_1^H,0)$ reaches $R-C_1^L$, the firm will increase both $\tau^B(C_1^H,0)$ and $\tau^B(C_1^L,0)$ simultaneously such that she can economize on the deadweight cost. If we suppose that $\tau^B(C_1^H,0)$ is lower than $R-C_1^L$, we get from the break-even condition of the counterparty that

$$\theta[\pi\tau^S(C_1^H,0)+(1-\pi)\tau^S(C_1^H,1)]+(1-\theta)[\pi\tau^S(C_1^L,0)+(1-\pi)\tau^S(C_1^L,1)]=\delta\tau^B(C_1^L,0), \tag{A9}$$

$$\Leftrightarrow \tau^B(C_1^H,1)=-\frac{[(1-\theta)\pi+\delta](R-C_1^L)-(1-\theta)(1-\pi)C_1^L}{\theta\pi}. \tag{A10}$$

This expression is actually lower than $R-C_1^L$ when

$$-\frac{[(1-\theta)\pi+\delta](R-C_1^L)-(1-\theta)(1-\pi)C_1^L}{\theta\pi}<R-C_1^L, \tag{A11}$$

$$\Leftrightarrow \frac{1+\delta-(1-\pi)\theta}{\pi+\delta}<\frac{R}{C_1^L}.\text{[32]} \tag{A12}$$

On the other hand, if $\tau^B(C_1^H,0)$ is greater than $R-C_1^L$, we can use $\tau^B(C_1^L,0)=\tau^B(C_1^H,0)$ and rearrange the break-even condition of the counterparty, (A9), to get

$$\tau^B(C_1^H,0)=\frac{(1-\theta)(1-\pi)C_1^L}{\pi+\delta}. \tag{A14}$$

We find that this expression is actually greater than $R-C_1^L$ when the reversal of (31) holds true, i.e.

$$\frac{1+\delta-(1-\pi)\theta}{\pi+\delta}\geq\frac{R}{C_1^L}. \tag{A15}$$

This establishes the Lemma.

□

32 Nonetheless, $\tau^B(C_1^H,0)$ can still be greater than zero. This is the case if

$$\frac{(1-\theta)+\delta}{(1-\theta)\pi+\delta}\geq\frac{R}{C_1^L}, \tag{A13}$$

i.e. if the shortfall in the firm's balance sheet in the low cash flow state is not too large.

A7 Proof of Proposition 5

Because the creditor correctly anticipates the behaviour of the firm, the creditor still breaks even. Hence, the firm's profits and the social surplus coincide again. The general form of the profits is analogous to before

$$\Pi = \theta(C_1^H + C_2 - R + \pi\tau^B(C_1^H, 0)) + (1-\theta)\pi(C_1^L + C_2 - R + \tau^B(C_1^L, 0)). \tag{A16}$$

From Lemma 3, we can read the transfers. If (31) holds true, the profits amount to

$$\Pi = \theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta(\frac{F}{\theta + (1-\theta)\pi} - C_1^L). \tag{A17}$$

Compared to the profits without hedging, the firm chooses to use the derivative when it promises higher profits, i.e.

$$\Pi > \theta(C_1^H + C_2) + (1-\theta)C_1^L - F \tag{A18}$$

which leads to

$$(1-\theta)\pi C_2 - \delta(\frac{F}{\theta + (1-\theta)\pi} - C_1^L) > 0. \tag{A19}$$

If (31) does not hold true, the profits are

$$\Pi = \theta C_1^H + (1-\theta)C_1^L + [\theta + (1-\theta)\pi]C_2 - F - \delta\frac{(1-\theta)(1-\pi)}{\pi + \delta}C_1^L. \tag{A20}$$

Then, the efficiency condition can be calculated as before and yields

$$(1-\theta)\pi C_2 - \delta\frac{(1-\theta)(1-\pi)}{\pi + \delta}C_1^L > 0. \tag{A21}$$

At last, note that in both cases the profits are strictly lower than in the case of no commitment.

□

A8 Proof of Lemma 4

As we have argued above, we will derive the equilibrium with $\tau^B(Z^H, 1, 1)$ and $\tau^B(Z^L, 1, 1)$ as well as $\tau^B(Z^H, 0, 1)$ and $\tau^B(Z^L, 0, 1)$ being equal.

The firm wants to set both $\tau^B(Z^H, 0, 0)$ and $\tau^B(Z^L, 0, 0)$ greater than or equal to $R - C_1^L$ in order to continue operations after C_1^L if the counterparty stays solvent. For optimality, these two transfers will also be equal due to the cost associated with the derivative contract. Then, the situation is essentially the same as in the case of no basis risk with the constraint that $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^L, 0)$ must be equal. The result follows immediately from Lemma 2.

□

A9 Proof of Lemma 5

The proof is similar to the case of no basis risk with the constraint that $\tau^B(C_1^H, 0)$ and $\tau^B(C_1^L, 0)$ must be equal and $\tau^B(C_1^H, 1)$ is not necessarily zero.

□

A10 Proof of Proposition 6

The derivative contract with close-out netting and commitment ensures that the firm can always continue after C_1^L if the counterparty stays solvent. Moreover, the deadweight cost are reduced to a necessary minimum because both $\tau^B(Z^H, 0, 0)$ and $\tau^B(Z^L, 0, 0)$ do not exeed $R - C_1^L$ and, under commitment, the creditor is not expropriated. Now, we show that both basic derivative contracts and contracts with close-out netting but no commitment lead to lower social surplus.

As we have argued, the basic derivative does not allow the firm to always continue after C_1^L given that the counterparty stays solvent. Hence, the firm looses some expected surplus. Moreover, the contractual repayment will be larger due to the additional default state. This increases also the deadweight cost $R - C_1^L$. All in all, basic derivative contracts lead to strictly lower social surplus than a contract with close-out netting and commitment.

A contract with close-out netting but no commitment leads to a lower social surplus because the firm will shift risk to the creditor, namely by expropriating the creditor in the states $(Z^H, 1, 1)$ and $(Z^L, 1, 1)$, which leads to greater deadweight cost.

$\square$